ALTERNATIVE STRATEGIES FOR SOLVING SUBTRACTION PROBLEMS GENERATED BY THIRD GRADE STUDENTS PROFICIENT WITH THE STANDARD ALGORITHM

by
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ABSTRACT:

This study explores third grade students' mathematical understandings by examining the strategies they generate to solve subtraction word problems. Ten students proficient with the standard subtraction and addition algorithms were individually observed and interviewed while using as many different methods as they could to solve three multi-digit subtraction and addition word problems. The students were found to rely excessively on algorithms and class-taught methods, even when inappropriate, and were divided in the sophistication of their modeling strategies, with half only using direct modeling with ones, while four successfully modeled with grouping. The creativity of the strategies used by some students, the prevalence of direct modeling with ones, and the misconceptions of many students provide insight into ways elementary teachers can promote students' present and future mathematical success.

Table of Contents

Section 1: ***Introduction***

Mathematics is an essential human pursuit, important for both practical and aesthetic reasons, and one for which understanding is crucial. Skills in mathematical reasoning are necessary for access to professional opportunities, personal fulfillment, and the performance of everyday tasks. Despite the importance of numeracy, mathematics assessments from the state to international levels indicate that students in the U.S. are deficient in their understanding of even basic concepts (National Research Council, 2001).

Although its importance is widely affirmed, understanding in mathematics is difficult to define, measure, and improve. Educational philosophers and researchers have highlighted the difference between procedural and conceptual understanding, as well as their interrelationship. Because critical thinking at the elementary level prepares students for success in more advanced mathematics, elementary mathematics educators should be careful not to teach students merely to recall and perform procedures without sound knowledge of the reasons behind these procedures. Policymakers in our nation recognize this need, endeavoring to amend the wide topic coverage but lack of depth prevalent in

U.S. teaching and undertaking the purpose of equipping students for higher-level reasoning (NCTM, 2006).

One major skill related to understanding, specified in both literature and national policies, is flexibility, the knowledge of multiple solution procedures for problems and the ability to adapt procedures to problems appropriately (Star, 2013). Flexibility is closely related to other mathematical capacities such as number sense (Yilmaz, 2017). Although there is a significant body research on strategies students use to solve math problems (Selter, Prediger, Nührenbörger, & Hußmann, 2001; Torbeyns, De Smedt, Ghesquière, & Verschaffel, 2009; Rumiati & Wright, 2010; Peltenburg, Heuvel-Panhuizen, & Robitzsch, 2012), little research has involved students coming up with different strategies to solve the same problems. Additionally, most studies done on student-invented strategies in addition and subtraction have been with students who have yet to learn the standard algorithm, often in classroom settings that emphasize invented strategies as part of the learning process (Thompson, 1994; Carpenter, Franke, Jacobs, Fennema, & Empson, 1998).

Thus, the aim of this research is to find out more about the generation of alternative strategies by students who are proficient with the subtraction algorithm and have been taught in a traditional elementary classroom. In this study, third grade students selected for proficiency in the subtraction and addition algorithms will be asked to solve multi-digit subtraction and addition word problems in as many different ways as they are able, providing insight into their thought processes and the sophistication of their understandings of number concepts.

The question asked by this study is: What strategies will students who are proficient with the standard subtraction algorithm generate to solve multi-digit subtraction word problems if asked to come up with as many strategies as they can? What will these strategies suggest about their mathematical flexibilities and knowledge of the logical bases of multi-digit arithmetic? The answers revealed will suggest ways educators can improve their teaching.

Section 2: ***Literature Review***

Researchers in mathematics education have often distinguished between mathematical proficiency that is superficial and that which has more depth. Evidence shows that students in the United States are skilled in one of these domains, performing computational procedures, while they are insufficient in their understanding of concepts, application, and problem-solving (National Research Council, 2001).

Discussion of these two aspects of mathematical proficiency began as early as the 1970s, where Skemp (1976) brought the difference between “instrumental” and “relational” understanding into scrutiny. “Instrumental” understanding, Skemp posited, is the knowledge of rules in mathematics; “relational” understanding is knowing both the rules and the reasons behind them. In a similar vein, researchers have distinguished procedural knowledge, “action sequences for solving problems,” from conceptual knowledge, “understanding of the principles that govern a domain and of the interrelations between pieces of knowledge in a domain” (Son, 2016, p. 109). Cauley (1986) also introduced the idea of “logical” knowledge, which she differentiates from conceptual knowledge in the context of the traditional subtraction algorithm in that while students with conceptual knowledge know the goal of subtraction is to take the bottom from the top number, students with logical knowledge understand that numbers can be composed using separate parts and retain their original value. In this thesis, “conceptual understanding” will refer to both logical and conceptual knowledge.

Research indicates that depth understanding is a necessity if students are to succeed in mathematics. Algorithms and other procedures learned in school are often forgotten by adulthood, with adults opting instead to use their own practical strategies to solve problems they encounter (Thompson, 1994). Because of this, it is essential that these adults have the depth of mathematical understanding to invent these strategies, which must be both effective and efficient. Studies have warned about the dangers of having only procedural knowledge, which can lead to loss of independent thinking and over-zealous applications when wrong or unnecessary, while the standard algorithms practiced so carefully in the classroom are, ironically, not used outside the classroom (Clarke, 2005). Not only can algorithms encourage students to relinquish independent thinking, they can actually counteract what they already know about place value (Kamii & Dominick, 2009). Teaching of the vertical addition and subtraction algorithms also curb students’ use of mental strategies (Yang & Huang, 2014).

Both teachers and students sometimes find it more rewarding to focus on procedural knowledge. Some students are only interested in how to get the correct answers, as the immediacy of this reward can boost self-confidence more easily. Teachers also need students to get the right answers quickly, especially when their large schedules encourage a shallow coverage of a wide variety of topics, for some of which a deeper understanding may be difficult to help students reach (Skemp, 1976). In the long term, however, it is better for students to know fewer principles and their relationships, with the ability to apply them flexibly, than to be bogged down by a large amount of rules

(National Research Council, 2001). Additionally, a deeper understanding of mathematics drives students to learn on their own, as mathematics becomes a worthy intellectual activity in its own right. On the other hand, a lack of understanding causes a fear and loathing of mathematics that is so prevalent among the general population (Skemp, 1976).

Though it is clear that procedural understanding can be present without conceptual understanding, the typical relationship between the two is complex. Students who are procedurally proficient generally have higher levels of conceptual knowledge. However, the knowledge of such students is still often limited in depth and accuracy. For example, many students generally proficient in subtraction have misconceptions about place value, such as the belief that the minuend is not the same after regrouping (Cauley, 1988). Students who are instructed both procedurally and conceptually make gains in achievement, but the gains are greater in students who are conceptually instructed, and conceptual instruction has a larger impact on improving procedural ability than the opposite. In addition, students who receive conceptual instruction are able to transfer their skills to new problems and generate multiple procedures (Rittle-Johnson & Alibali, 1999). It can be concluded that it is important to help children build conceptual understanding in fundamental areas of mathematics such as multi-digit addition and subtraction.

Progression of Children's Understanding of Number Concepts

The concept of number is the central focus of mathematics in the primary grades (National Research Council, 2001). For elementary students, understanding of base-ten concepts and operations is a stepping stone to more advanced math skills (McGuire, Kinzie, Kilday, & Whittaker, 2010). According to the National Council of Teachers of Mathematics (2006), second grade students should be able to use place value ideas to write, compare, order, compose, decompose, and create equivalent representations of multi-digit numbers. According to Richland, Begolli, & Näslund-Hadley (2017, p. 14), working on number sense in the early grades means "developing an understanding of the base ten pattern and place value, and using these for arithmetic calculations of addition, subtraction, multiplication and division with increasingly large numbers".

How do children progress from their natural understanding of numbers to facility with breaking apart and rearranging multi-digit numbers? Babies have an innate neurological system for discerning shape and magnitude, even before they can use language or interpret symbols (Richland et al., 2017). By the time children reach kindergarten, they can solve addition, subtraction, multiplication, and division problems effectively by modeling with physical objects (Carpenter, Fennema, Franke, Levi, & Empson, 2000) or using counting strategies (National Research Council, 2001). However, many students encounter a hurdle when the base-ten system system is introduced; in fact, this is when procedural proficiency and conceptual understanding in young children begin to diverge (National Research Council, 2001). A particular danger at this point is that students begin to think of math merely as a practice of memorizing disconnected rules, procedures, and computations (Richland et al., 2017).

It is not that these children do not have the necessary foundations for learning the base-ten system; rudimentary understanding of two-digit place value is present in even pre-k students (McGuire et al., 2010). However, students are often thrust into using algorithms without preparation or explanation (Ma, 2013). Studies have shown that students follow a general trajectory as they come to understand the basis of adding and subtracting multi-digit numbers. Cognitively Guided Instruction (CGI) focuses on students' developmental thinking and is successful in significantly improving student achievement as compared to control classes (Carpenter et al., 2000). Students observed in this type of program have given insight into how children naturally develop an understanding of numbers and operations (Carpenter, Fennema, Franke, Levi, & Empson, 2014).

The most basic strategy students start out using to solve problems is *direct modeling*, using objects to represent the actions in the problems. As children solve more problems, they develop more abstract strategies such as *counting* from one quantity to the other to find the difference. They also begin to use *number facts*, such as separating 8 + 5, an unknown problem, to 8 + 2 + 3 = 10 + 3 = 13. As students start solving multi-digit problems, they similarly model using base-ten materials and progress to use the number strategies they know to invent their own algorithms (Carpenter et al., 2014).

Teaching Subtraction for Understanding

Teaching subtraction is an effort that involves extensive knowledge and a deep understanding of mathematics on the part of the teacher (Ma, 1999). It is essential for teachers to intentionally provide opportunities for children to become familiar with the part-whole nature of numbers (Cauley, 1986).

Cognitively Guided Instruction

CGI is one approach that is enormously successful (Carpenter et al., 2014). A strength of this approach is that students are allowed to reason through problems and invent their own strategies to add and subtract. Research shows that children who invent strategies before being exposed to standard algorithms have superior knowledge of base-ten concepts, are more capable of extending their knowledge to new situations, and make fewer systematic errors. By contrast, the majority of children who learn the algorithm first generally do not use invented strategies, and many are unable to invent their own strategies when asked (Carpenter et al., 1998). Encouraging students' spontaneous ways of solving problems is more effective than strategies commonly taught in the classroom, such as the key-word method, which often relies on superficial aspects of word problems (Carpenter & Moser, 1984).

Schema-Based Instruction

Schematic tools have also been shown by research to be useful. A schematic diagram is "a visual tool to assist students in paring down information so that only the important structural information remains" (Van Klinken, 2012, p. 5). The main benefit of this tool is improving word problem comprehension, reducing confusion and misinterpretation by equipping students to find the structures of problems rather than being distracted by surface-level details (Boonen, Reed, Schoonenboom, & Jolles, 2016).

Schema-based interventions with addition and subtraction word problems help students both improve their performance and maintain what they have learned, more so than traditional classroom teaching methods (Jitendra, Griffin, Leh, Adams, & Kaduvettoor, 2007; Jitendra & Hoff, 1995; Jitendra, Hoff, & Beck, 1997).

The Role of Manipulatives

While a schematic diagram is a visual representation of the action in a problem, another important tool for solving problems is a visual representation of the quantities themselves. These representations help students see the relationships between numbers, enabling them to better understand or generate strategies. The number line, for example, can stimulate students to use different strategies such as direct subtraction, indirect addition, and indirect subtraction to solve subtraction problems in ways that make sense to them (Murdiyani, Zulkardi, Putri, Van Erde, & Van Galen, 2013).

Manipulatives are often used in classrooms as visual and physical tools. Instruction with manipulatives has a greater effect on student learning than instruction with only abstract symbols (Carbonneau, Marley, & Selig, 2013). One common manipulative used to help students work with multi-digit quantities is base-ten blocks. Students instructed using base-ten blocks perform considerably better on tasks related to understanding of place value in subtraction than those instructed with traditional methods. There is a positive effect even when the blocks themselves are not present: For children familiar with base-ten blocks, thinking about them while working helps them self-correct written errors and remain aware of place value (Fuson & Briars, 1990).

Despite all of their benefits, manipulatives alone do not ensure learning; for instance, sometimes students do not connect their manipulations with the ideas behind them (Clements & McMillen, 1996). In order for children to benefit from using them, concrete materials must connect with their current understandings (Rivera Vega, 1996). Manipulatives with noticeable physical qualities can distract children from their inherent meaning in a task (Carbonneau et al., 2013). Computer-based manipulatives can remediate some of the negative effects of physical ones, such as distractibility (Clements & McMillen, 1996), but even with virtual manipulatives, teachers must read the instructional suggestions for them in order to successfully apply them to their classroom teaching (Yuan, 2009). Since manipulatives can differ in their effectiveness due to various factors, such as characteristics of the student population, amount of teacher guidance, and the topics they are used with, it can be easy to misapply them (Carbonneau et al., 2013).

It is crucial for teachers to make clear the relationships between manipulatives and the mathematical ideas they are conveying, something which not every teacher is able to do (Ma, 1999). Teachers with extensive training have notable results when teaching with manipulatives, while this may not be the case for other teachers (Sowell, 1998). Even self-reflective teachers with hard-working, conscientious students can struggle to bridge the gap between physically trading ones and tens rods and the concept of regrouping with two-digit numbers (Schifter, Bastable, & Russell, 1999).

Education systems in mathematically successful countries utilize careful systems of guiding students from concrete materials to abstract numbers (Zhou & Peverly, 2005). A type of this approach is concreteness fading, a three-step progression from the concrete to the increasingly abstract that has shown potential for success (Fyfe, McNeil, Son, & Goldstone, 2014). Studies suggest that the concrete-representational-abstract sequence is successful in teaching students subtraction with regrouping (Flores, 2009; Mancl, Miller, & Kennedy, 2012).

Ultimately, when using any kind of instructional aid, it is important to help students reach their own understandings. Research indicates that one of the main factors impacting learners' success in solving math word problems is their ability to use metacognition and self-regulation (Vula, Avdyli, Berisha, Saqipi, & Elezi, 2017).

Multiple Solution Methods as an Indicator of Understanding

Awareness of the importance of conceptual understanding and the challenges educators face in helping students achieve it motivates ways to evaluate understanding. Concrete assessments of mathematical cognition can be difficult (Ma, 2013). The intent of this study is to investigate elements of students' cognition through their own created methods of solving problems.

The ability to generate and apply multiple solution methods is a common theme in mathematics education literature. Baroody & Dowker (2003) contrast the more procedure-based routine expertise, the performance of school-learned exercises quickly and accurately, with adaptive expertise, the flexible and creative application of understood procedures, such as the invention of procedures when solving new problems, a skill dependent on the integration of conceptual and procedural knowledge. The National Research Council (2001) emphasizes *flexibility*, which is the ability to choose appropriate procedures for certain problems and adjust them in variable conditions. This skill is characterized both by the knowledge of multiple solution procedures and the ability to select appropriate procedures for particular kinds of problems.

National recommendations for the teaching and learning of two-digit addition and subtraction associate flexibility closely with conceptual understanding. The *Curriculum and Evaluation Standards for School Mathematics* of the National Council of Teachers of Mathematics (1989, as cited in Rivera Vega, 1996) recommends that addition and subtraction computations focus on the understanding of concepts and defines understanding in mathematics as the ability to create and connect multiple representations of mathematical ideas. The Common Core State Standards (2010) specifies that students be able to add and subtract two-digit numbers and solve word problems involving two-digit addition and subtraction using objects, drawings, strategies, and equations.

Many studies show that flexibility and the invention of strategies are indicative of a substantial understanding in mathematics (Baroody & Dowker, 2003). Moreover, successful students can adapt a variety of strategies to previously un-encountered problems, represent mathematical situations in different ways, and explain the connections between different representations (National Research Council, 2001). Carpenter et al. (1998) propose that the use of invented strategies is a direct indicator of the level of understanding students have of base-ten concepts.

In the same light, encouraging flexibility and inventiveness improves student performance and understanding. Many countries with successful mathematics education programs hold a philosophy that students "should actively engage in learning by finding all possible solutions to a problem, providing rationales for their solutions, and comparing and contrasting various ways of solving a problem and, based on their evaluation, determining the most effective solution" (Zhou, 2005, p. 271). Having exposure to alternative algorithms and opportunities to discuss these and come up with their own algorithms helps students think critically at an early age (Jong, Dowty, Hume, & Miller, 2016; Schifter et al., 1999). Students increase their number sense as they use a variety of solution strategies for operations (Yilmaz, 2017). Studies suggest that teaching a mixture of methods in subtraction, rather than a single method, works best for achievement and retention (Sawyer, 1973). Including informal methods in teaching can motivate students, boosting students' confidence in their ability to solve similar problems independently (Große, 2014).

There is a correlation between the type of instruction students receive and the flexibility of their problem solving. Students instructed in classes oriented towards investigation and problem-solving are able to apply diverse strategies to different types of multi-digit addition and subtractions problems, while those instructed in more skills-oriented classes use the standard strategies they were taught on all types of problems (Torbeyns, 2009). Students can grow in skill and experience yet never develop the ability to invent strategies if they are not specifically taught or encouraged to do so (Torbeyns, 2009). Even for students who used invented strategies, they can almost disappear after the introduction of standard algorithms, even in cases where they would be more efficient; it unknown whether these strategies are forgotten or merely displaced by an routine procedure (Selter, 2001). It is therefore important to encourage students to always think critically. The inclination toward a traditional method displayed by many preservice teachers can impede their ability to connect their own reasoning with student-invented methods and thus lead to the discouragement of student thinking (Son, 2016).

It is hoped that this study will provide information on the degrees of base-ten conceptual understanding that students in a traditional school environment have and probe into the relationships and possible inconsistencies between their procedural and conceptual knowledge. Trends in student strategies recorded in this study can also reveal clues about the type of teaching they have received and offer recommendations for teachers seeking to improve their instruction in this area.

Section 3: ***Research Methodology***

Participants and Setting

Participants were third-grade students at Della Davidson Elementary School in Oxford, Mississippi. At the time the study was conducted, according to the math teacher, the students already knew subtraction with regrouping, which had been heavily taught in second grade and was retaught for mastery in third grade because many students were not yet proficient. Strategies used included the standard algorithm, partial differences, the number line, the CUBE strategy (similar to keyword strategy), and word problems without numbers where students could draw pictures (similar to schema strategy). Manipulatives used include base-ten blocks, tiles, and counters.

All students in the two classes the researcher was student teaching were initially eligible for the study. Students were selected for proficiency in the standard algorithm after completing a sheet of 10 subtraction and 2 addition problems, each consisting of a three-digit number in the hundreds followed by a 2-digit number (see Appendix A). Those who missed either zero problems or one problem were considered proficient. Both student consent and parental permission for the study were also obtained. Students who were proficient, expressed consent for the study, and whose parents expressed consent for the study were selected to participate in the performance task. Ten students were in this group (6 female and 4 male, age range 8-9).

Research Task

The performance task for each student was done within school settings during the non-instruction class period (50 minutes per day in which students would read, work on intervention material, or go to the enrichment program). Time taken for the task ranged from 10-45 minutes, depending on the student's working pace and willingness to keep thinking.

Students were selected one by one to do the research task. During the task, the researcher sat facing the student at a table in an area with few distractions. Pencils and unlimited notebook paper were provided. The student was given a sheet with three word problems, each featuring two numbers, one 3-digit number in the hundreds and one 2-digit number. The problems were from one of two sets of approximately equal difficulty (see Appendix B), and which set the student got was random. As studies have found that difficulty levels can vary with types of word problems (Artut, 2015), the researcher ensured there was a wide variety of problem types. The problems in Set 1 were Separate Start Unknown, Compare Referent Unknown, and Join Change Unknown, and the problems in Set 2 were Compare Quantity Unknown, Separate Change Unknown, and Join Start Unknown, respectively, as classified by Carpenter et al. (2014). In each set, the first problem was directly solvable using addition, while the next two were directly solvable using subtraction, ensuring that the students would not simply guess which operation to use.

The researcher introduced the task by saying, “I’m going to ask you to solve three word problems. You can solve them in whatever way you want. After you do each one, I will ask you to explain how you did it. I will also ask you to do it using different ways if you can.” For each problem, the student could choose whether to let the researcher read the problem out loud or read it independently. The student then worked on the first problem. After the student got an answer, the student explained his/her method. The researcher then asked, “Can you think of another way to do it?” or, “If you couldn’t do it this way, how would you do it?” If the student was stuck, the researcher used the prompt, “Can you draw a picture?” The process would continue until the student could not think of another method, at which time the student would begin the next problem. For methods which were taking a long time, such as directly modeling with ones, the student was allowed to simply explain what he/she would do. During the task, the researcher took electronic notes about the student’s methods and explanations on her laptop.

No answers or hints were given while a student was working. However the researcher gave minor assistance if she believed it would yield further insight into the student’s thought processes or encourage creativity. Examples of this include reminding Student 6 about his initial method in Problem 3 and asking Student 7 to reread Problem 2 after he miscopied a number (details in Findings).

Data Collection and Analysis

The students' identities were not recorded. Instead, each student was given a number from 1 to 10.

Data was analyzed by examining each of the students' methods and coding them as follows:

SA - Use of the correct standard algorithm

DMO - Direct modeling with ones

DMG:T - Direct modeling with groups of ten

DMG:O - Direct modeling with groups of another quantity

C - Counting, in the context of addition

C:D - Counting down, in the context of subtraction

C:U - Counting up, in the context of subtraction

FF - Use of "fact families" (relationship between operation and its inverse)

N - Use of a number strategy, such as partial differences

E:WA - Use of the wrong algorithm (addition instead of subtraction or vice versa)

E:M/D - Erroneous use of a multiplication or division strategy

E:O - Another erroneous strategy (used in only one case, in which the student suggested a two-step problem)

In the rare case that a student used two different approaches during one solution process (happened one time), the solution was counted as two methods and coded separately.

Each of the students' methods was also coded according to its success in reaching the correct answer as follows:

(S) - Successful (the correct answer was reached)

(U) - Unsuccessful (the correct answer was not reached)

(I) - Incomplete (used only for DMG:T and DMG:O strategies in which the student had a correct thought process until getting stuck on regrouping quantities)

Note: SA (U) and E:WA (U) are differentiated in that SA (U) refers to the use of the algorithm appropriate to the problem with an error in computation, while E:WA (U) refers to the use of an addition or subtraction algorithm inappropriate to the problem.

After each method was coded, data was calculated on the strategies the students used, and trends were analyzed for potential significance.

Section 4: ***Findings***

Notes on Student Work

The following table (**Table 1**) is a record of all methods used by all students for each problem, along with notes clarifying their usage. Each note corresponds to the method directly across from it. Notes in quotations are in the students' own words.

Table 1

Student	Strategies	Notes
1	Problem 1 ● SA (S) ● DMO (S) ● E:M/D (U) Problem 2 ● E:WA (U) ● DMO (U) Problem 3 ● SA (S) ● DMO (S)	 draw 181 circles, draw 66 more circles "do it in a rectangle and use the area" used addition instead of subtraction used addition instead of subtraction draw 113 circles and mark out 29
2	Problem 1 ● SA (S) ● DMO (S)	 used large circles to separate dots counted

	Problem 2	
	• SA (S)	
	• C:D (S)	count down 53 using fingers from 110
	Problem 3	
	• SA (S)	
	• DMO (S)	draw 119 pencils, cross out 49, and count the rest
3	Problem 1	
	• SA (S)	
	• E:M/D (U)	student expressed thinking of “multiplying”
	• E:WA (U)	used standard algorithm subtracting 66
	• E:M/D (U)	“usually in math we would draw big circles and we would put an amount of dots in it and add them all together, and that would usually be your answer”
	Problem 2	
	• SA (S)	used standard algorithm adding 66
	• E:WA (U)	tried to subtract 111 from 197
	• E:WA (U)	wrote out standard algorithm without operation and
	• E:WA (U)	thought for a long time
	Problem 3	
	• SA (S)	used standard algorithm to add 29 “but it was the wrong answer”
	• E:WA (U)	
	• E:M/D (U)	draw 29 large circles, putting one dot down in each circle while counting to 113, see how many dots are in a circle and subtract that when done
4	Problem 1	
	• SA (S)	

	• FF (U) • DMG:O (S) Problem 2 • SA (S) • FF (U) • DMG:T (S) Problem 3 • N (S) • SA (S) • DMG:T (S)	“what minus 144 = 59” drew tally marks, with each tally representing 5 and “0.8 of a tally” representing 4 “53 plus what = 110” drew different symbols for 10, 3, and 1 horizontal: 119 - 49 = 70, noting that 9 - 9 = 0 used symbols from problem 2, adding symbols for 100 and 9
5	Problem 1 • SA (S) • E:WA (U) • E:M/D (U) • DMO (S) Problem 2 • SA (S) • DMO (S) Problem 3 • SA (S) • DMO (S) • C:U (S)	 used standard algorithm to subtract 181 from 247 “area model” draw 181 circles and add 66 and count draw 197 circles and cross out 86 draw 113 circles and cross out 29 “have 29 dollars and keep adding numbers until you get to 113”
6	Problem 1 • SA (S) • SA (S) • DMO (S)	 reversed order of addends however, stated “wouldn’t think that I would draw adding because 144 is too much to draw”

	Problem 2	
	• SA (S)	
	• E:M/D (U)	“110 divided by blank = 53”
	• FF (U)	53 + ___ = 110
	Problem 3	
	• SA (U)	regrouped when unnecessary, getting 60
	• DMG:T (U)	made 119 using symbols for hundreds, tens, and ones, then added 4 more tens
	• DMG:T (I)	after being reminded subtraction was done at first and asked if he could think of a way to do subtraction with the symbols, stated, “I would have to take away 4 tens and I didn’t have 4 tens” (referring to only having one ten symbol)
7	Problem 1	
	• SA (S)	wrote 181 + 66 = 247 horizontally, but based on explanation, it was done in the same way as the standard algorithm
	• FF (U)	blank - 181 = 66
	• SA (S)	66 + 181 = 247
	• DMO (S)	66 dots plus 181 dots
	• DMG:O (I)	drew “people” (stick figures); each stick figure represented 2 people; “it was a lot of people; it would be faster”; however, switched to using one figure to represent one person because “if you kept on adding 2, it would go over 181”
	Problem 2	
	• E:WA (U)	written horizontally, added instead of subtracting, misread 86 as 68, also added incorrectly: 197 + 68 = 283
	• SA (S)	done after being asked to reread the problem; written horizontally, did correct operation but wrote plus sign

		instead of minus
	• DMO (S)	draw 197 squares, cross out 86
	• E:O (U)	“Could it be a 2 step problem? 197 + 86 and then… actually that wouldn’t work.”
	• DMG:O (I)	“One page could stand for a certain amount. Would you have to do 197 divided by something to find out your number?”
	Problem 3	
	• SA (S)	written horizontally at first but switched to vertical algorithm
	• DMO (S)	draw 113 dollars, cross out 29 and find how many are left
	• DMG:O (I)	“Could every dollar represent 3 dollars? 56*2 = 112. So that wouldn’t work.”
8	Problem 1	
	• SA (S)	
	• E:WA (U)	tried to subtract 144 from 203
	• DMO (S)	draw 59 and 144 circles and add them
	Problem 2	
	• SA (S)	
	• DMG:T (S)	drew 11 “10’s” (rectangles), crossed out 6 rectangles, and wrote 7
	Problem 3	
	• SA (S)	
	• DMG:T (S)	drew 11 rectangles, crossed out 4, “realized you could count by 10’s”
9	Problem 1	
	• SA (S)	
	• C (S)	start at 181 and count 182, etc.
	• SA (S)	adding using standard algorithm in her head
	• DMG:O (S)	“kid” (stick figure) represents 20, half a “kid” represents 1

	Problem 2 ● E:WA (U) ● E:WA (U) ● C (U) ● C (U) ● DMG:O (U) Problem 3 ● SA (S) ● SA (S) ● C:D (S) ● DMG:O (S), N (S)	 used standard algorithm in her head (addition instead of subtraction) written standard algorithm (addition) on fingers on a number line drew whole circle for 50, half for 40, quarter for 1; re-counted from start used standard algorithm in her head written standard algorithm Drew 100 dollar bill, 10 dollar bill, and "3 dollar bill". Used mixture of partial differences and decomposition. Procedure was as follows: -cross out 3, 26 dollars left -cross out 10, 16 dollars left -split 100 into 50 and 50 -take 16 from 50 and get 34 -add 34 to 50 and get 84
10	Problem 1 ● E:WA (U) ● C:U (U) ● DMO (U) Problem 2 ● SA (S) ● N (S) ● DMO (S) Problem 3 ● SA (S) ● N (S) ● DMO (S)	 subtracted instead of adding draw 59 circles and add up to 144 "what plus 3 = 10 and what plus 5 = 10" with circles add 60 to 49 and 10 more, which would be 70 with circles

General Findings

The average number of methods each student used per question was 3.3. Throughout all the problems, each student came up with an average of 3.9 unique strategy types. On average, each student used 2.8 unique successful strategy types and got 2.6 questions correct with at least one strategy.

Frequencies of Strategy Types

Algorithms (~45.7% of all strategies). With the exception of one student for question (Student 4, Question 3, for whom SA was the second strategy), all students used a standard algorithm first for every question. In only one instance, the algorithm was the correct operation but calculated incorrectly (SA (U)). About 30.2% of all algorithm strategies used the wrong operation (addition instead of subtraction or vice versa) (E:WA). 8 out of the 10 students used the wrong operation at some point.

Direct modeling with ones (~17.0% of all strategies). This strategy had a relatively high success rate, with only about 12.4% being unsuccessful. Every unsuccessful instance was due to the student using the wrong operation, as frequently happened with algorithms.

Direct modeling with grouping (~12.8% of all strategies). Only about 53.6% of DMG strategies were successful. However, about 30.8% of all DMG strategies were incomplete, meaning the student had a productive line of thought until becoming confused about how to exchange values using the strategy. The remaining percentage of DMG strategies were unsuccessful, again due to adding instead of subtracting or vice versa. About half of the DMG strategies used tens (DMG:T), while the others used a different grouping, such as 5's or 3's (DMG:O).

Counting (~7.5% of all strategies). Counting was successful about 57.3% of the time. Again, in every unsuccessful instance, the student used the incorrect operation. Students who used counting (4 out of 10) seemed to prefer one of counting down (C:D) or counting up (C:U) over the other; no students used both.

Multiplication/division mix-ups (~6.4% of all strategies). Four out of the 10 students exhibited this type of strategy at least once. Examples include using a rectangle's area (Student 1), directly expressing the intent to multiply (Student 3), drawing circles and putting dots inside (Student 3), using an area model (Student 5), and directly expressing the intent to divide (Student 6).

Number strategies (~4.3% of all strategies). Every instance of the use of number strategies was successful. 3 out of the 10 students used a number strategy at least once. The number strategies used were place value (Students 4 and 10), partial differences (Student 9), incremental addition (Student 10), and decomposition (Student 9).

Fact families / inverses (~4.3% of all strategies). Three students referred to the relationship between an operation and its inverse (commonly called fact families in their math class) as a method for solving a problem at least once. None were able to arrive at the correct solution using these; however, the relationships described by these students were all correct.

See **Fig. 1** for a visual representation of the frequency and success rate of each type of strategy used by the students. Note that "correct" in this graph refers to successful strategies, while "incorrect" refers to both unsuccessful strategies and incomplete strategies, the latter of which were not technically incorrect; for example, the majority of "incorrect" direct modeling with grouping strategies were incomplete.

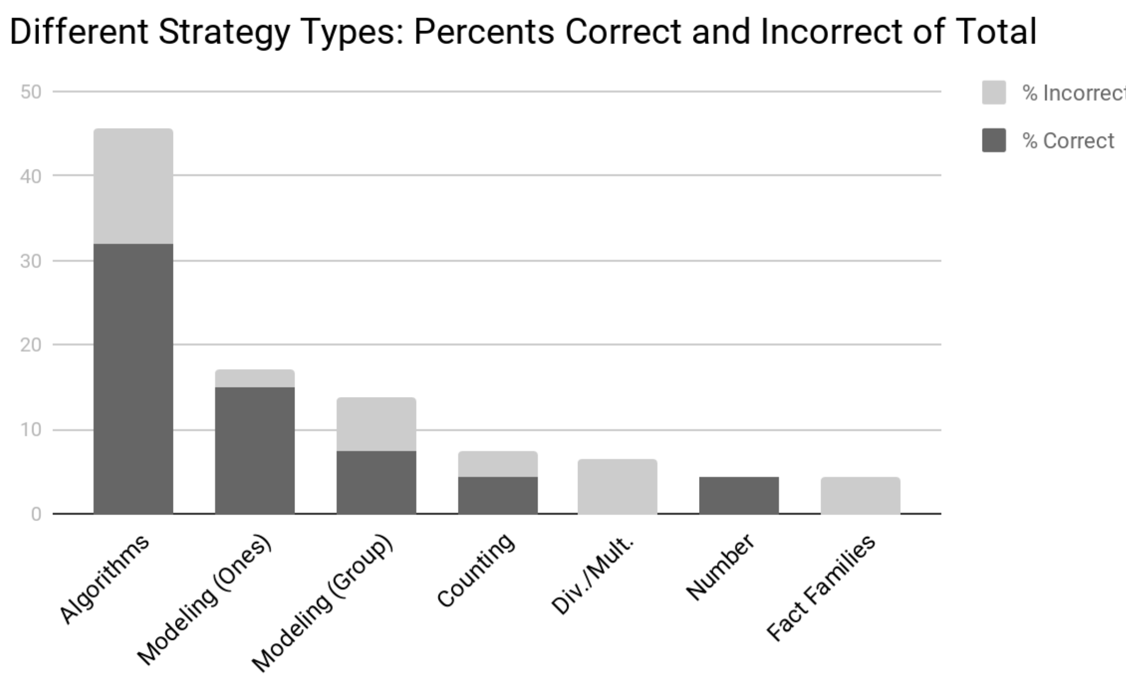

Fig. 1

Differences Between Students

Modeling Strategies

The students were divided in the types of strategies they demonstrated success with. One student (Student 3) had success in the *standard algorithm only*. Six students had success with *both the standard algorithm and direct modeling with ones*; two of the six (Students 6 and 7) attempted direct modeling with grouping but did not succeed (DMG:I). Three students (Students 4, 8, and 9) had success with *the standard algorithm and direct modeling with grouping*, one of whom also used direct modeling with ones successfully (the other two did not use direct modeling with ones at all).

Development of Reasoning

The student used increasingly creative strategies as they progressed from the first to the second and third problems. Four students came up with a new successful strategy in Problem 2 that they did not use previously, and three did so in Problem 3, although the number of strategies did not increase (from 35 to 32 to 29 total in Problems 1, 2, and 3, respectively); the students seemed to abandon ineffective strategies when they found more efficient ones.

One of the most significant insights the students had was that of modeling by grouping quantities greater than one. Five students (Students 1, 2, 3, 5, 10) had no such insights and used the standard algorithm, direct modeling with ones, and/or counting only. Student 5 is unique in this group in that he did have a moment of insight regarding using counting up in Problem 3. Two students (Students 4 and 9) also did not have observable insights about direct modeling with grouping but instead used this strategy successfully from the first problem. Both these students did, however, reach smaller insights while inventing and modifying their methods. One student (Student 7) thought of using direct modeling with grouping throughout all three problems but never consistently became stuck. Two students (Students 6 and 8), perhaps most excitingly, discovered (or rediscovered) direct modeling with grouping while working on Problem 2 and subsequently used this strategy instead of direct modeling with ones.

Factors Related to Success Rate

Success rate for each student was calculated by finding the percentage of successful methods out of all methods the student used. The students who used direct modeling with grouping did not have a greater success rate (63.3% to 65.8% otherwise), contrary to the idea that ability to use more advanced strategies correlates with better overall performance. There was a negative correlation between the overall number of strategies generated by a student and success rate (see **Fig. 2**). However, there was a slight positive correlation between the number of unique strategy types generated by a student and success rate (see **Fig. 3**), suggesting that struggling students are more likely to generate methods of the same type, while successful students are more flexible in their generation of strategies. In any case, success rate may not be an accurate measure of mathematical ability because students who choose methods they are comfortable with, regardless of their sophistication, will have higher success rates, while students who take risks may have lower success rates.

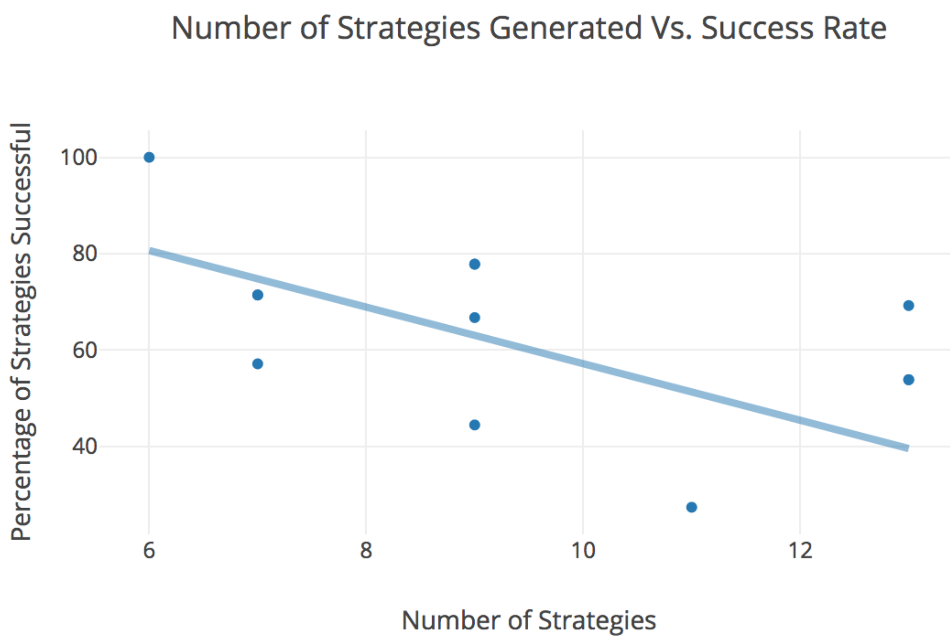
Number of Strategies Generated Vs. Success Rate
Percentage of Strategies Successful
100
80
60
40
6
8
10
12
Number of Strategies

Fig. 2

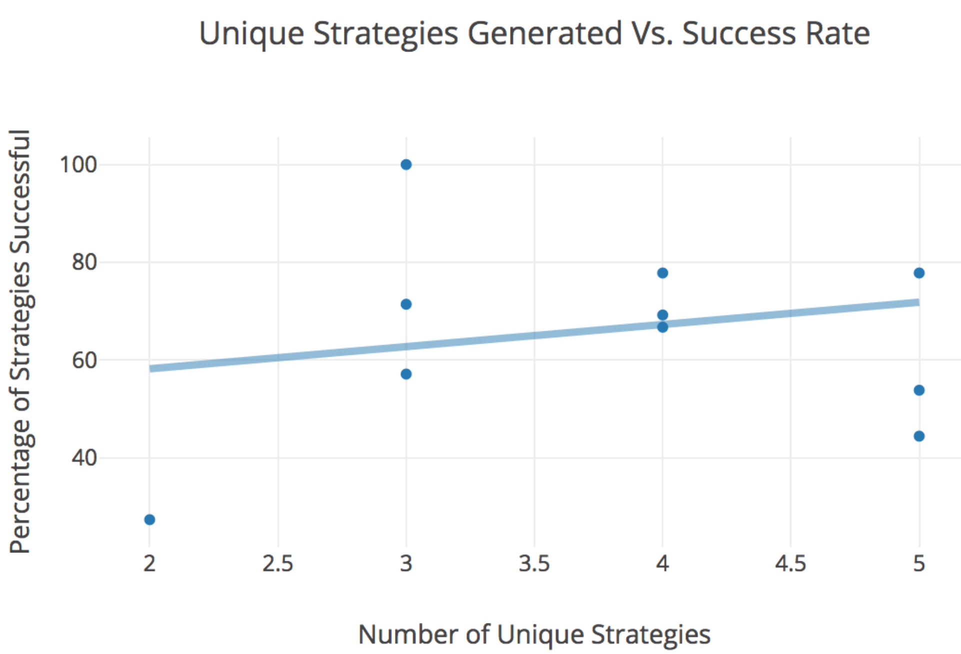
Unique Strategies Generated Vs. Success Rate
Percentage of Strategies Successful
100
80
60
40
2
2.5
3
3.5
4
4.5
5
Number of Unique Strategies

Fig. 3

Discussion

Many of the results from this study were surprising. The following are trends that emerged from the data applicable to most or all of the students.

Reliance on Superficial Procedures

Although the researcher anticipated some use of incorrect operations, it was unexpected that they comprised such a large percentage of student strategies, for instance, of all algorithms used around 30.2%. In some cases, the student may have misread or not comprehended the question, but no student spent a long time reading a problem; rather, students began working immediately after reading a problem, suggesting an emphasis on calculation rather than problem comprehension in the student's instruction. Another possibility for the use of an incorrect operation is that the student did not know which operation to use. Again, however, no student spent a significant amount of time deliberating which operation to use. It is unknown whether the students felt pressured to get an answer or did not find using the correct operation important.

It is known, however, that in an unsettlingly large amount of cases (occurred with Students 1, 3, 5, 6, 7, and 8), a student used the correct operation at first in a problem, only to suggest one or more incorrect operations as other strategies to solve the same problem. It is unlikely that the students' use of correct operations were coincidences because in most cases, they knew when to use addition and when to use subtraction in their first try. This suggests that the students intuitively understand the nature of the quantities and relationships in a problem but may shut down this understanding when focusing on "how to do it". Additionally, this suggests that, to a certain degree, the students have not solidified their conscious knowledge of the different purposes of adding, subtracting, multiplying, dividing, or that these operations usually yield different results. A notable example of this is the student (Student 3) who, in an attempt after using the standard algorithm successfully, stated, "Usually in math we would draw big circles and we would put an amount of dots in it and add them all together, and that would usually be your answer," (referring to a division strategy used in class). This confusion may have been partly caused by an overemphasis on procedures during class or perhaps an overabundance of procedures used in class, leading to a shortage of in-depth discussion of their meanings.

Emerging Intuition of Regrouping

Because it is known that the students used base-ten blocks during class (although it is not known exactly how often), it is surprising that (a) so many students did not model with groups at all or struggled while attempting to model with groups (b) half the students who modeled with groups did not use groups of ten. The researcher hypothesizes that either the base-ten blocks were not used often enough to make a lasting impression on most of the students or their use did not follow the progression of student understanding and was taught more procedurally.

In any case, overall the students appeared to have a medium-level understanding of base-ten concepts and regrouping, lagging behind their ability to compute answers algorithmically as well as their ability to intuit whether to add or subtract. Many students used only the simplistic, painstaking strategy of directly modeling with ones, drawing individual dots or circles even up to the hundreds. Only half even attempted to model with groups, with even fewer using any number strategies.

For the three students who successfully modeled with groups, there were many positive surprises. In every case of direct modeling with grouping, the strategy did not seem to be a memorized procedure. In some cases, creative symbols were used to represent groups of quantities; Student 4 even used the fact that 0.8 of 5 was 4. Student 9's final strategy in Problem 3 is an excellent example of an invented strategy and demonstrates the flexibility of the student's understanding of numbers. Successful grouping strategies showed that the depth of proficiency these students have is with not only the standard algorithm but also the principles behind the algorithm, the composition and decomposition of numbers.

Section 5: ***Conclusions***

It can be inferred from this study that students who are proficient with the standard algorithm are at a variety of different levels in the sophistication of their modeling strategies and their understanding of base-ten concepts. Although they can fluidly perform the procedure of an algorithm with regrouping, they may, may not, or may partially understand the number concepts that comprise regrouping.

Results from this study also suggest that the ability to generate multiple unique strategies is correlated with stronger overall performance. However, the use of more advanced strategies was not found to be related to overall performance; more research is needed confirm and/or determine the reasons for this.

Additionally, if the findings in this study are indicative of the broader population, then students spend too little attention on problem comprehension and the problem story and too much attention on the performance of superficial answer-supplying procedures, often while neglecting their intuitive mathematical understandings.

Finally, this study suggests that students are capable of reaching their own insights about regrouping and other numerical properties while working on problems, and thinking of ways to do problems other than the standard algorithm enormously benefits them.

Limitations and Suggestions for Further Research

A major limitation of this study is the small sample size. More students, possibly from different classes, schools, and grade levels, are needed to determine whether the problem-solving trends observed in this study are typical and whether they evolve with age.

Situational factors may also have influenced the results in this study. In a more informal situation, such as when solving real-life problem at home, at a park, or at a store, the students may have used more informal methods. Also, the researcher's language was significantly process-oriented (i.e. "do the problem"). It would be useful to see if the students preferred less superficial methods if the researcher reworded the task to be more comprehension-oriented.

Although difficulty thinking of using grouping strategies or manipulating modeled grouped quantities exhibited by many students hints at their level of understanding of base-ten concepts, it is unknown exactly where their understandings fall and what they do and do not know. Asking the students questions about, for example, what happens to the numbers during regrouping in the standard algorithm, could provide more information.

It should also be mentioned that during the time of the study, the students were learning division and multiplication in their math class. Doing the research task at a different time of year would probably reduce the amount of erroneous multiplication and division strategies.

Finally, students who were not proficient with the standard algorithm were not included in the study. It would be of great interest to observe whether these students exhibit superior, equal, or lesser ability to use strategies other than the standard algorithm and how their understanding of base-ten concepts compares with the understanding of students who are proficient with the standard algorithm.

Implications for Teaching

This study shows the benefits of doing a similar performance task with students to assess their facility with multi-digit numbers. The students' problem-solving methods provide ample guidelines for how to further help these and similar students. For example, the next step for the children who are modeling with ones is to guide them towards the idea of modeling with groups, while for those just beginning to model with groups but getting stuck, it would be helpful to guide them towards how to exchange their groups with ones, their hundreds with tens, etc. Finally, for the students who successfully modeled with groups, moving forward would be about extending the variety of grouping strategies they can use and connecting their pictorial modeling processes with the standard algorithm or other algorithms.

As many students exhibited a contradictory combination of knowing at first which operations to use yet suggesting wrong ones later, this study suggests that an increased focus on problem comprehension and a smaller focus on superficial procedures would reduce errors by bolstering rather than suppressing students' intuitive understanding of what is going on in a word problem.

It is also evident from this study that teachers need to provide more opportunities for students to use their own strategies. Instruction with manipulatives needs to be deliberate in its connections to student thinking. Most importantly, teachers should believe in and encourage their students' ability to learn by critical thinking, invent new strategies, and discover mathematical truths.

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Appendix A

119 - 45	158 - 29	175 - 53	173 + 59
181 - 76	197 - 85	139 - 86	144 - 25
126 + 97	110 - 12	113 - 66	199 - 49

Appendix B

Set 1

1. Many kids were at the park. Then, 181 kids went back home. Now there are only 66 at the park. How many kids were at the park at first?
2. Your book has 197 pages. Your book has 86 more pages than your friend's book. How many pages does your friend's book have?
3. You are saving money to buy a present. The present costs 113 dollars. You have 29 dollars so far. How many more dollars do you need to buy the present?

Set 2

1. 59 kids are car riders today. There are 144 more bus riders than car riders today. How many kids are bus riders?
2. 110 kids were on a field trip at the farm. Then, some kids went home with their parents. Now there are only 53 kids at the farm. How many kids went home with their parents?
3. There were some pencils in a box. The teacher put 49 more pencils in the box. Now there are 119 pencils in the box. How many pencils were in the box at first?